Random Assignment with Optional Participation

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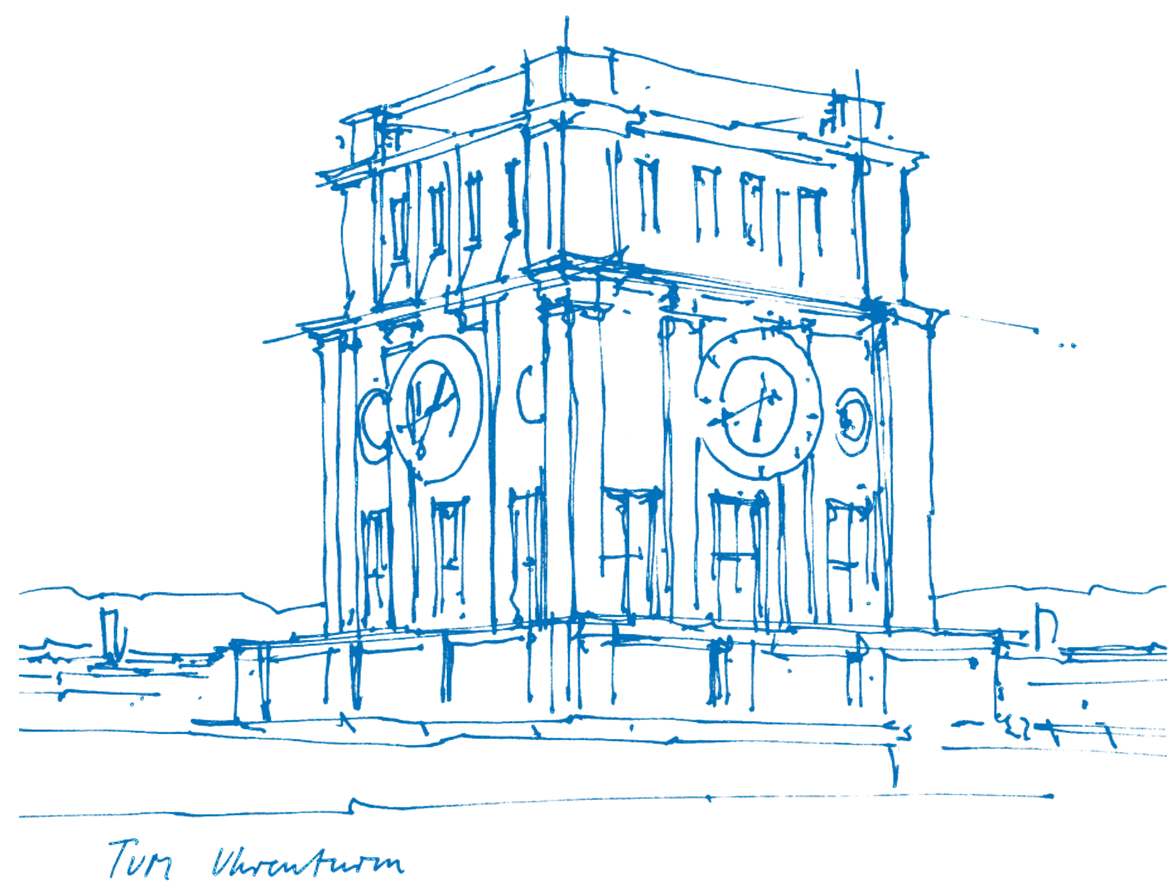
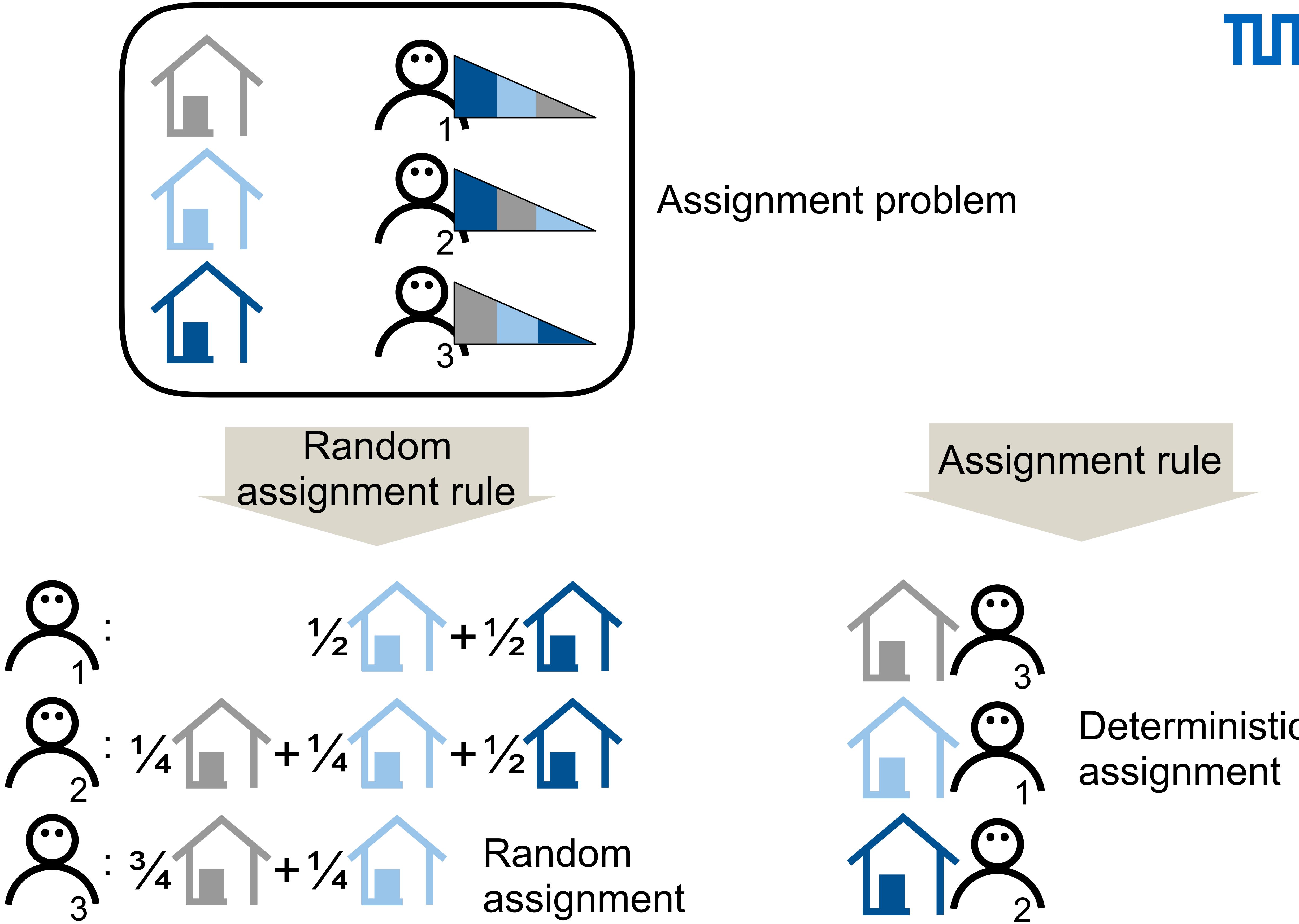
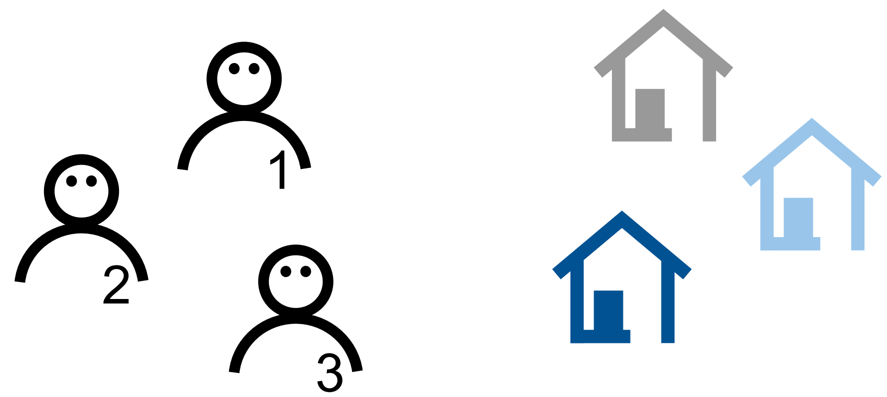
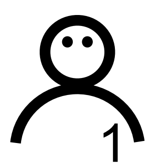


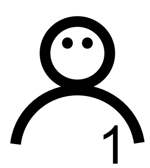

1
2
3
Assignment problem
Random assignment rule
Assignment rule
1: ½ + ½
2: ¼ + ¼ + ½
3: ¾ + ¼
Random assignment
3
1
2
Deterministic assignment



- A company wants to allocate offices to workers
  - everybody has to receive something

- Determining / submitting one’s preferences is associated with effort / cost

- Allocation of  if he participates: ½  + ⅙  + ⅓ 

- Allocation of  if he abstains: 

- According to stochastic dominance (SD) ½  + ⅙  + ⅓  > 

- Is there always an incentive to participate? (i) How to model participation?
  (ii) How to define incentives?

Modeling participation

Two ways to define abstention:

indifference — absent agents treated as if they were completely indifferent

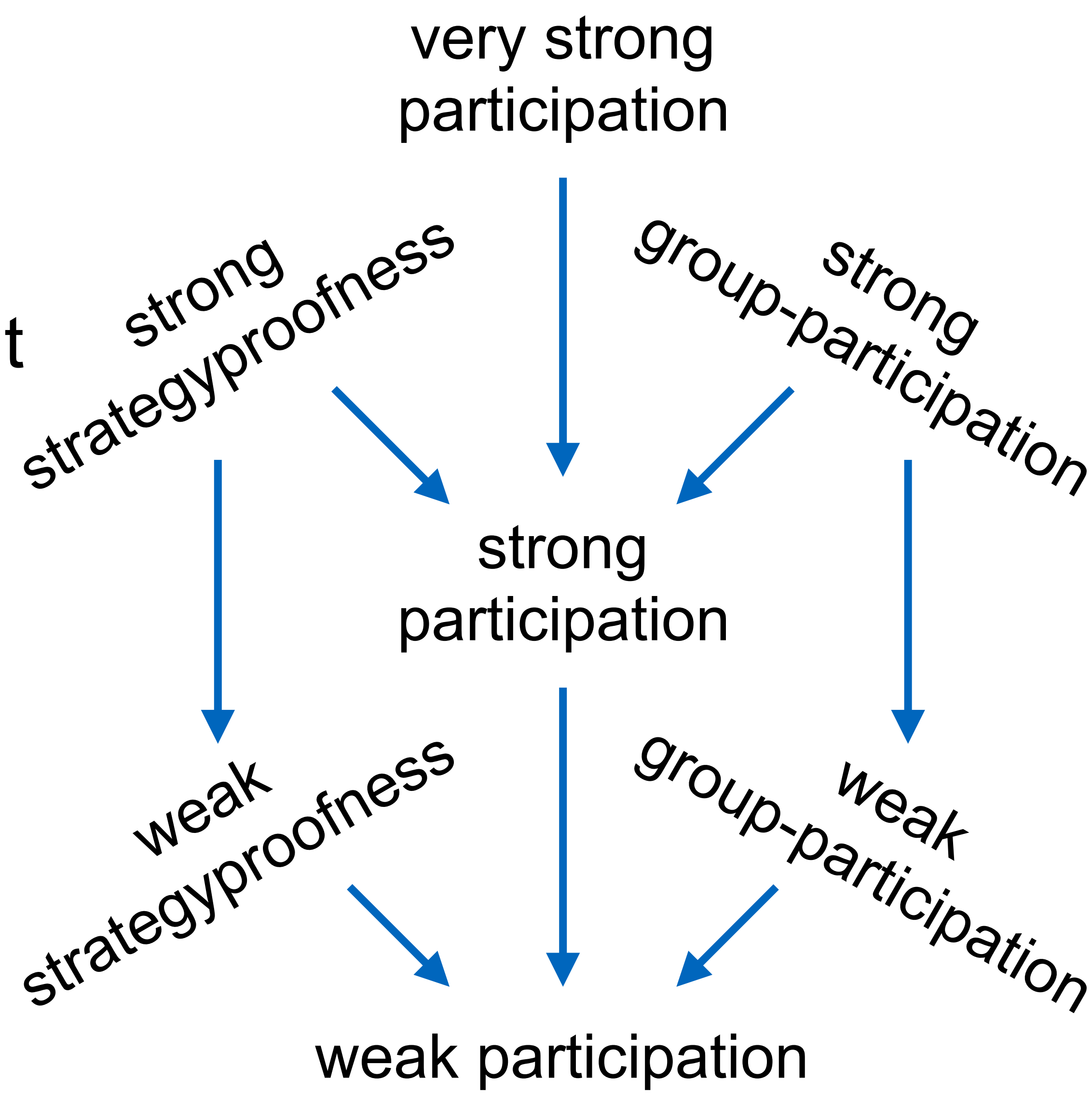
two rounds
(1) participating agents get probability share 1 according to preferences
(2) remaining probability shares distributed uniformly to absent agents

➔ both interpretations equivalent for the random assignment rules considered

Defining incentives

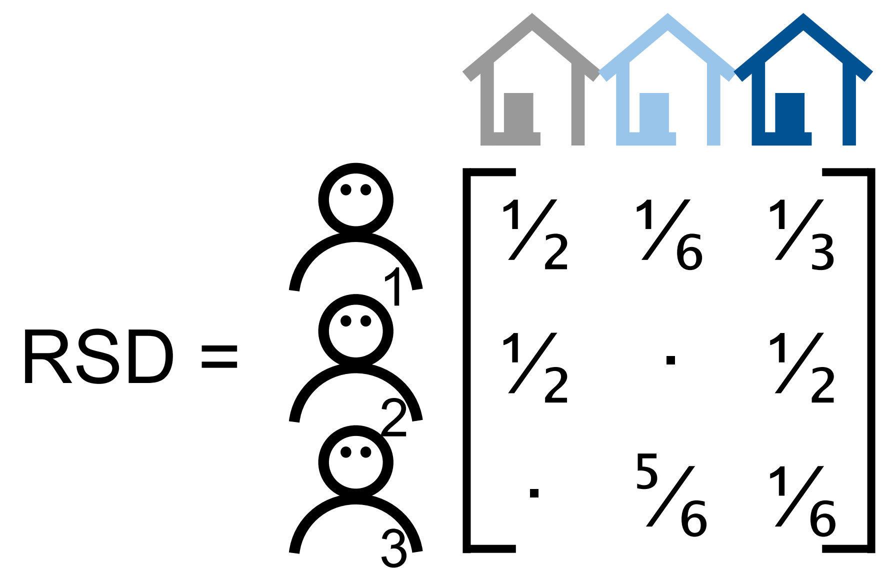
Three notions of SD-participation [Brandl et al., 2015]:

weak participation	participating cannot yield a worse assignment
strong participation	participating always yields a (weakly) better assignment
very strong participation	participating always yields a strictly better assignment



Random serial dictatorship (RSD)

- **Assignment rule**
  - select a sequence of agents uniformly at random
  - in this order, every agent picks his most preferred remaining house
- **Participation**
  - RSD satisfies very strong participation
  - RSD satisfies strong group-participation



(Extended) Probabilistic Serial (PS)

- **Assignment rule**
  - each agent reserves probability of his first indifference class of houses at uniform speed
  - when a set of houses is completely reserved, agents continue with their most preferred still available indifference class

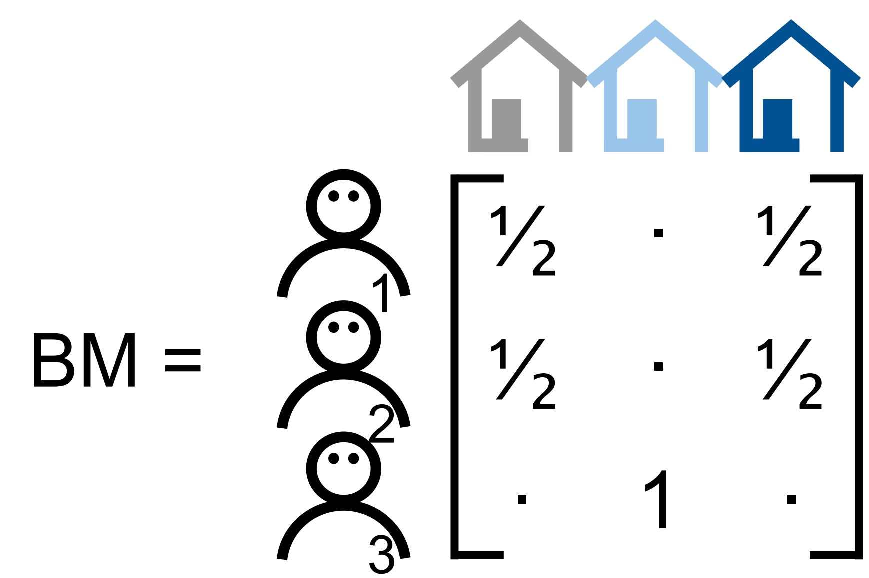
- **Participation**
  - PS satisfies very strong participation
  - PS satisfies strong group-participation

$$PS = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & \cdot & \frac{1}{2} \\ \cdot & \frac{3}{4} & \frac{1}{4} \end{bmatrix}$$

(Naive) Boston Mechanism (BM)

- **Assignment rule**
  - in the *i*th round, every remaining house is assigned to an agent ranking it at *i*th place uniformly at random
  - all assigned agents and houses leave the procedure after each round

- **Participation**
  - BM satisfies very strong participation
  - BM does not satisfy strong group-participation
  - BM satisfies weak group-participation

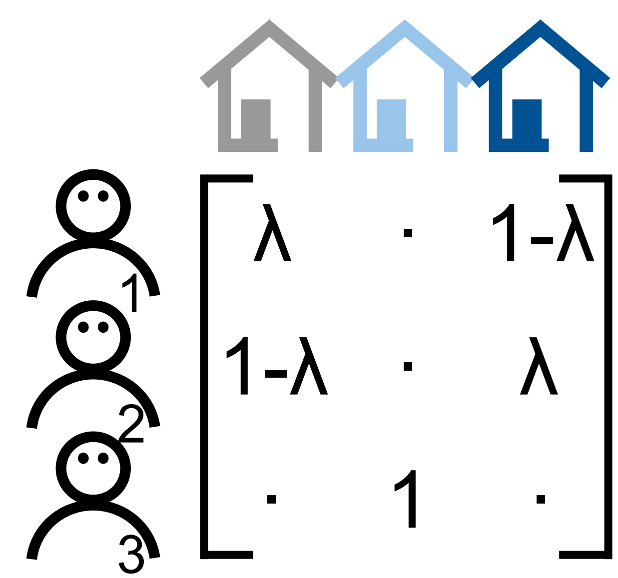


Popular random assignment rules (PRA)

- **Class of assignment rules**
  - random assignments, such that there does not exist another random assignment which is majority preferred
  - correspond to (probabilistic) weak Condorcet-winners in the majority graph

- **Participation**
  - PRA do not satisfy very strong participation
  - PRA do not satisfy strong group-participation
  - PRA satisfy weak group-participation
  - **Open problem**:
    Do PRA satisfy strong participation?

PRA =



$$\begin{bmatrix} \lambda & \cdot & 1-\lambda \\ 1-\lambda & \cdot & \lambda \\ \cdot & 1 & \cdot \end{bmatrix}$$

Overview & conclusion

	Very strong participation	Strong group-participation	Weak group-participation
Random Serial Dictatorship	✅	✅	✅
Probabilistic Serial	✅	✅	✅
Boston Mechanism	✅	❌	✅
Popular random assignment rules	❌	❌	✅

- Results largely positive; participating (basically) always incentivized